Math 347 Worksheet

Lecture 20: Properties of sequences and Cauchy sequences

October 19, 2018

I. Suppose that $x_n \to 0$ and that $|y_n| \leq 1$ for $n \in \mathbb{N}$.

(i) Find the flaw in the following computation for $\lim x_n y_n$.

$$\lim x_n y_n = \lim x_n \cdot \lim y_n = 0 \cdot \lim y_n = 0.$$

(ii) Find a valid proof of the fact that $\lim x_n y_n = 0$.

II. For the statements below, determine if they are true or false. If false provide a counter-example if true give a proof.

1) Let $(x_n)_{n\geq 1}$ be a sequence of real numbers.

(i) If $(x_n)_{n\geq 1}$ is unbounded, then $(x_n)_{n\geq 1}$ has no limit;

(ii) If $(x_n)_{n\geq 1}$ is not monotone, then $(x_n)_{n\geq 1}$ does not converge;

2) Suppose that $\lim x_n = L$.

(i) For all $\epsilon > 0$, there exists $n \in \mathbb{N}$, such that $|x_{n+1} - x_n| < \epsilon$;

(ii) There exists $n \in \mathbb{N}$ such that for all $\epsilon > 0$, $|x_{n+1} - x_n| < \epsilon$;

(iii) There exists $\epsilon > 0$ such that for all $n \in \mathbb{N}$, $|x_{n+1} - x_n| < \epsilon$;

(iv) For all $n \in \mathbb{N}$, there exists $\epsilon > 0$ such that $|x_{n+1} - x_n| < \epsilon$.

3) Consider $(x_n)_{n\geq 1}$ a sequence of real numbers.

(i) If $(x_n)_{n\geq 1}$ converges, then there exists $m \in \mathbb{N}$ such that $|x_{m+1} - x_m| < \frac{1}{2^m}$;

(ii) If $|x_{m+1} - x_m| < \frac{1}{2^m}$ for all $m \in \mathbb{N}$, then $(x_n)_{n\geq 1}$ converges.